(L-3.)

Properties of Composition of Relation.

Let R, S and T be relation on X then

(i) $(R \circ S) \circ T = R \circ (S \circ T)$

(ii) $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$

(iii) $R \circ (S \cap T) \subseteq (R \circ S) \cap (R \circ T)$

(iv) $(R \cup S) \circ T = (R \circ T) \cup (S \circ T)$

(v) $(R \cap S) \circ T \subseteq (R \circ T) \cap (S \circ T)$ (vi) $(R^{-1})^{-1} = R$

(vii) $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

* **Equivalance Relation** :- A relation R on a set A which is reflexive, symmetric & transitive is called equivalance Relation on A. and $a \in A$, then the subset $R_a = \{b \in A : (a, b) \in R\}$ is called the equivalance class of A modulo R determined by a.

Ex :- $R = \{(a, b) : a \leq b, a, b \in R\}$ is an equivalance relation

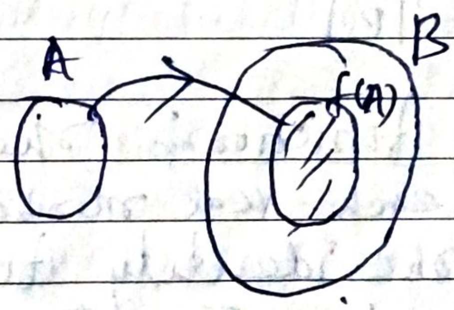
FUNCTION:- Let A and B be two sets. A subset of $A \times B$ is called a function or a mapping (or a map) from A to B. if

(1) $\forall\ a \in A$, $\exists\ b \in B$ such that $(a,b) \in F$
(2) $(a,b_1) \in F$ and $(a,b_2) \in F \Rightarrow b_1 = b_2$

NOTE:- If f is a function from a set A to B, then we write it as $f: A \rightarrow B$.

* **Domain, Co-domain and Range of a function**

Let $f: A \rightarrow B$ be a function, then the set A is called the domain, B is called co-domain and set of value of f is called the range of f and we write it $f(A)$ clearly $f(A) \subseteq B$.

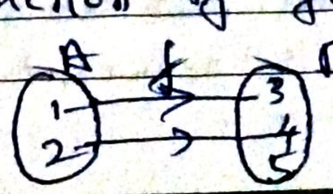


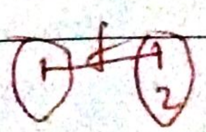
* **equality of two function:-** two function f and g are equal if

(i) domain of f = domain of g
(ii) $f(x) = g(x)$ $\forall\ x$ (image same हैं सभी x के लिए)

types of functions

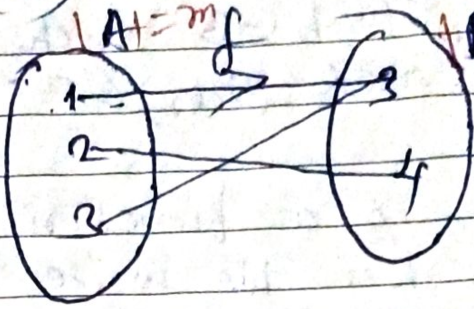
(1) **into function:-** A map $f: A \rightarrow B$ is called an into function if $f(A)$ is proper subset of B.





(2) Onto (surjective) function :- A map $f : A \to B$ is called surjective map or onto map if $\forall\, b \in B\ \exists\ a \in A$ such that $(a, b) \in f$ or $f(a) = b$.

$|A| = m$ $\quad |B| = n$



$\sum_{r=1}^{n} (-1)^{n-r}\, {}^{n}C_{r}\, (r)^m$

(3) Constant mapping :- If K is a fixed real number then $f(x) = K\ \forall x \in \mathbb{R}$ is called a constant function.

* Remark :- The total number of constant function from a set A to set B is $card(B) = |B|$

(4) Identity function :- The function f that assign each real number to itself is called the identity function. It is denoted by $I : \mathbb{R} \to \mathbb{R}$. Hence the identity function $I : \mathbb{R} \to \mathbb{R}$ is defined as

$$I(x) = x \quad \forall x$$

(5) One-one (injective) function :- A map $f : A \to B$ is called injective or 1-1 if

$$f(a_1) = f(a_2) \Rightarrow a_1 = a_2$$

OR, equivalently,

$$\text{if } a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$$

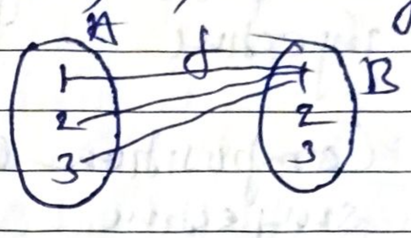
If $|A| = m$, $|B| = n$, $m \le n$

* 1-1 function $= {}^{n}P_{m} = \dfrac{\lfloor n}{\lfloor n-m}$

Date
Page

(6) Bijective function :- A mapping $f : A \to B$ is called Bijective if f is both 1-1 & onto.

(7) Many to one function :- if two or more distinct element in A have the same image i.e. if $f(x) = f(y) \Rightarrow x \neq y$ then f is called many one function.



(8) Inverse function :- Let $f : A \to B$ be an onto function, then $f^{-1} : B \to A$ which associates to element $y \in B$, the element $x \in A$ such that $f(x) = y$ is called an inverse function of $f : A \to B$.

(9) Composition of function :- Let $f : A \to B$ and $g : B \to C$, then composition of function f and g is a mapping from $A \to C$ given by mapping $gof : A \to C$ s.t.

$$(gof)(x) = g(f(x)). \ \forall x \in A.$$

evidently, domain of g is equal to the co-domain of f.

→ Properties of Composition mapping :-

(1) If $f : A \to B$ is a bijective function, then $fof^{-1} = I_B$ and $f^{-1}of = I_A$, where I_A and I_B are identity mapping of set A and B respectively

Date
Page

(2) If $f: A \to B$, $g: B \to C$ and $h: C \to D$ then
$$h \circ (g \circ f) = (h \circ g) \circ f.$$

(3) Inverse of a bijective map is bijective

(4) Composition of any two injective map is injective

(5) Composition of any two surjective map is surjective

(6) " " bijective " "

(7) Let $f: A \to B$ and $g: B \to C$ be maps then

(a) If $g \circ f$ is surjective $\Rightarrow$ g is surjective
" " " $\Rightarrow$ g is surjective

(b) If $g \circ f$ is injective $\Rightarrow$ f is injective
$g \circ f$ is 1-1 $\Rightarrow$ f is 1-1

(c) If $g \circ f$ is bijective $\Rightarrow$ g is surjective & f is injective

(9) Let $f: A \to B$ and $g: B \to C$ be bijective mapping then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(10) (a) A maps $f: A \to B$ is injective iff f can left cancelled in the sense
$$f \circ g = f \circ h \Rightarrow g = h$$

(b) A map $f: A \to B$ is surjective iff f can be right cancelled in the sense that
$$g \circ f = h \circ f \Rightarrow g = h$$

PAPERISTO

Result: (1) Let $f: X \to Y$ be a map and $A \subseteq X$ then $A \subseteq f^{-1}(f(A))$. Also, $A = f^{-1}(f(A))$ $\forall A \subseteq X$ if f is injective

(2) Let $f: X \to Y$ be a maps and $B \subseteq Y$ then $f(f^{-1}(A)) \subseteq B$. Also

$B = f(f^{-1}(A))$ $\forall B \subseteq Y$ if f is onto

(3) Let $f: X \to Y$ be a map. Let X_1 & X_2 be a subsets of X. then

$f(X_1 \cup X_2) = f(X_1) \cup f(X_2)$

(4) $f(X_1 \cap X_2) \subseteq f(X_1) \cap f(X_2)$, equality hold when f is 1-1

(5) Let $f: X \to Y$ be a map. Let Y_1 and Y_2 be subset of Y. then

(a) $f^{-1}(Y_1 \cap Y_2) = f^{-1}(Y_1) \cap f^{-1}(Y_2)$

(b) $f^{-1}(Y_1 \cup Y_2) = f^{-1}(Y_1) \cup f^{-1}(Y_2)$

(c) $f^{-1}(Y_1 - Y_2) = f^{-1}(Y_1) - f^{-1}(Y_2)$

EX:- 1.1

(1) If $|A \cap B| = 3$ then $|(A \times B) \cap (B \times A)| = \underline{3^2 = 9}$?

(2) If $|A \cap B| = m$ then $|(A \times B) \cap (B \times A)| = \underline{m^2}$?

(3) Show that surjective map need not be injective

(4) Show that injective map need not be surjective

(5) Show that there is no surjective map from a set A to its power set P(A)